(WORK SIX OF THE SEVEN PROBLEMS: TWENTY POINTS EACH)

1.) a.) Compute the determinant of $\mathbb{M} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & -2 & -5 \end{bmatrix}$

b.) $\mathbb{M}^{\mathbf{t}} =$

$\mathbb{A} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix}$ $\mathbb{B} = \begin{bmatrix} 1 & 4 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$ $\mathbb{C} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ $\mathbb{D} = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$

c.) Compute $\mathbb{A}\,\mathbb{B}$.

d.) Compute $\mathbb{C} + \mathbb{D}$.

2.) $|\mathbb{A}| = 4$ $|\mathbb{B}| = 2$ $|\mathbb{C}| = -1$

a.) $|\mathbb{A}\,\mathbb{B}| =$

b.) $|\mathbb{A}\,\mathbb{B}\,\mathbb{C}| =$

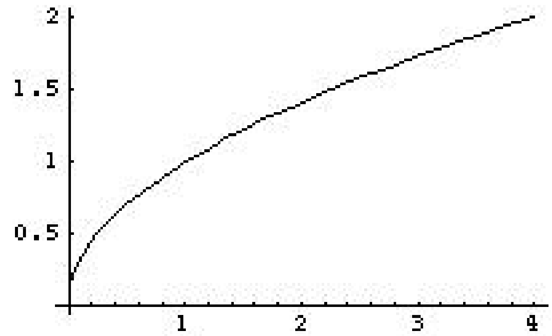
c.) $|\mathbb{A}\,\mathbb{B}^{-1}| =$

d.) If $\mathbb{B}$ is a 2 x 2 matrix, $|2\,\mathbb{B}| =$

e.) $|\mathbb{A}^{t}| =$

e.) Describe a method to compute the inverse of a matrix.

3.) For $\vec{F}(\vec{r}) = 3\ y\ \hat{i} + 6x^2 y\ \hat{j} + 12\ z\ \hat{k}$ consider the path integral $\int_{path} \vec{F}\ d\vec{r}$ from [0,0,0] to [4,2,0] along the path $y = \sqrt{x}$ in the *x*-*y* plane.



a.) Give a parameterization of the path with expressions for x, y, z, dx, dy and dz.

b.) Evaluate the integral.

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4.) The classic eigenvalue problem: $\mathbb{M}\,\vec{v} = \lambda\,\mathbb{1}\,\vec{v} = \lambda\,\vec{v}$

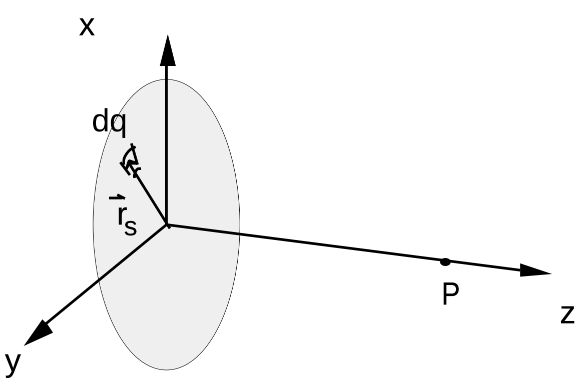
Given $\mathbb{M} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find the eigenvalues and eigenvectors.

5.) A thin circular disk with uniform surface charge density σ and radius *R* lies in the *x*-*y* plane centered on the origin. The goal is to compute the electric field due to the charge at points on the z axis. The following equation is used to compute the electric field due to the charge distribution.

$$\vec{E}(\vec{r}_P) = \int \frac{k\ dq}{r_{SP}^2}\hat{r}_{SP}$$

$$dq_s = \begin{cases} \lambda\ d\ell & \text{charge spread along a line} \\ \sigma\ dA & \text{charge spread over an area} \\ \rho\ dV & \text{charge spread throughout a volume} \end{cases}$$

a.) What are your choices for integration variables ?



b.) Give an expression for $\vec{r}_s$

$\vec{r}_s =$

c.) $\vec{r}_p =$

d.) $\vec{r}_{sp} =$

e.) $\hat{r}_{sp} =$

f.) Give the limits of integration for each integration variable.

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6.) The electric field is computed as the negative gradient of a scalar potential function. $\vec{E} = -\vec{\nabla} V(\vec{r})$. In spherical coordinates, $V(r,\theta,\phi) = A\, r^{-2} \cos\theta$.
Compute $\vec{E} = -\vec{\nabla} V(\vec{r})$.

7.) Given $\vec{F}(x,y,z) = \left(x + x\,y\right)\hat{i} - \left({}^{1}/{}_{2}\right) y^{2}\ \hat{j} + z\ \hat{k}$.

a.) Compute $\vec{\ }\ \vec{F}$.

b.) Compute $\oint \vec{F}\ \ \hat{n}\, dA$ by any method for the surface of a sphere of radius 2 centered at [1,1,1].

eXtra-Credit (not worth the trouble !) (**<u>ONLY</u> TWO POINTS EACH**)

X1.) Compute $\vec{E} = -\vec{\ } V(\vec{r})$ for $V(r,\theta,\phi) = A\ r^{-2}\ \cos\theta$ in Cartesian coordinates.

X2.) Compute the electrostatic potential due to a uniformly charged spherical shell. Let σ be the uniform surface charge density and R be the radius of the shell. Find $V(\vec{r})$ for points inside and outside the shell.

X3.) Give values for:

$\varepsilon_{2431} =$

$\sum_{i,j,m=1}^{3} \delta_{ij}\ \varepsilon_{mij} =$

$\sum_{j=1}^{3} \delta_{ij}\ \delta_{jm} =$

$\varepsilon_{2431765}\ \varepsilon_{2431756} =$